OPA version **3.39**

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1 Introduction

1.1 Status

OPA is under continuous development, but some colleagues all over the world like to use it, therefore these instructions have been written.

Incomplete features or known problems are described using italics.

1.2 General information

OPA's main purpose is to support the development of electron (positron) storage rings. Emphasis is on visualization and interactivity. OPA is in particular useful for designing high brightness light source lattices, but may be used for transfer lines and other types of lattices as well. Storage ring design with OPA starts from scratch and ends at a bare (i.e. error free) lattice with optimized 2D dynamic apertures, to be passed on to other codes like TRACY or MAD, which use more complete models.

1.3 History and acknowledgements

OPA is based on the code OPTIK from Klaus Wille (DELTA/Dortmund), who started in the 80's already to work on a design tool for electron rings. In 1993 he kindly passed it on to the author, who developed it further and heavily used it for the design of the Swiss Light Source, SLS. Algorithms for sextupole optimization and signal processing were contributed by Johan Bengtsson (now BNL). Simon Leemann (MAX-Lab) did a lot of tests and suggested many extensions and changes during the design of MAX-IV. Michael Borland (APS) tested the module for non-linear optimization for consistency with the ELEGANT code and helped to find several bugs.

1.4 Capabilities and limitations

OPA includes the following features:

- Human readable lattice input/output and robust lattice reader.

- Text editor and modular editor for elements and segments.
- Interactive graphical design of linear optics with many convenient functions as knobs, zoom, matching, tune diagram etc.
- Calculation of momentum dependent lattice parameters.
- Interactive optimization of sextupole and octupole Hamiltonian in 1^{st} and 2^{nd} order.
- Tracking with FFT: Poincar plots, resonance guesses and amplitude dependent tunes.
- Tracking of dynamic apertures.
- Tracking of Touschek lifetime.
- Display of lattice geometry.
- Orbit distortion and correction.
- Calculation of magnet currents and export of EPICS snap files.
- *Export of TRACY, MAD-X and ELEGANT lattice files.*
- Several ways to export graphics: image snapshots, export as `*.wmf` file, *export of* `*.gp` *file for GNU-plot*
- Export of data in text files.

OPA has the following limitations:

- Only correct for rings with large circumference and high energy:
  - High relativistic approximation: $\gamma \gg 1$
  - Paraxial approximation: $x = dx/ds \ll 1$
  - Large curvature approximation: $x \ll \rho$ (bending magnet radius)
- Exactly correct only on energy (i.e. $\Delta p/p = 0$), since it uses internally x' instead of the canonical coordinate p_x.
- No betatron coupling and no skew elements included, i.e. it allows only flat lattices.
- Nonlinear elements are treated by a 2^{nd} order symplectic integrator only.
- Only planer insertion devices allowed. Simple model as dipole array.
- Only 4D tracking (no synchrotron oscillations).
- Approximative (not exact) calculation of path length and dipole down-feed from higher multipoles (contribution to radiation integrals).

2 How to use

2.1 Installation and files

OPA is a Windows program, best tested for XP. There is no special installation procedure: OPA is one single executable `opa.exe` and does not need additional libraries or files to run.

Table 1: Files read or written by OPA

file	read	write	
files in the OPA folder:			
`opa3_path.ini`	•	•	remember last used files
`[diagopa.txt]`		•	[code development]
files in each data folder:			
`opa3_set.ini`	•	•	user settings for this folder
`(name).opa`	•	•	lattice files
`(name)_(plot).wmf`		•	plot export as Windows metafile
`(name)_(plot).txt`		•	plot data export as text file
`(name)_(plot).out`		•	plot data export as text file
`(name)_(plot).gp`		•	GNU plot command file for plotting `*.out`
`(name).lat`		•	TRACY lattice file
`(name).mad`		•	MAD-X lattice file
for EPICS export of magnet currents (in data folder):			
`allocation.opa`	•		power supply allocation
`calibration.opa`	•		magnet calibrations
`(name).snap`		•	EPICS snapshot file
`[(name).dev]`		•	[outdated]

So all to do is to create a folder for OPA and perhaps one or another for the data. Note, that OPA needs write permission also for the folder where it is located, so it does not run from a CD. Table 1 gives an overview of files read or written by OPA.

2.2 Start

The OPA menubar gives access to all functions. The large buttons on the GUI access only the most used functions. If functions are not available, because a prerequisite is missing (e.g. periodic solution for tracking) the buttons are disabled and shown in gray.

"Active file" shows the selected lattice file (`*.opa`). A lattice file contains a list of **elements** (magnets) and a list of **segments**. A segment is a series of elements and segments (see section 3 for definition of elements and segments).

The "active segment" is selected from the segment list on the GUI. From this segment, the **lattice** is built by recursive expansion of the segments to get the elements line-up, which is shown in the message window when pressing "show lattice".

Upon start only the "File" and "Edit" options in the menu bar and the corresponding buttons are enabled. One may either read a `*.opa` file or create a new one by using one of the two editors.

2.3 Editor

There is the OPA editor and the text editor. New users may prefer the OPA editor, advanced users the text editor.

The OPA Editor shows a list of elements and of segments, and further has input fields for beam energy, element apertures and for a comment text. The functionality is quite obvious and does not need much explanation:

The element editor allows to define the type of element and to set its parameters (see sec.3.3 for element properties).

The segment editor allows to type in the line-up of elements. Insert and Delete keys may be used. Undefined elements are shown in red, before saving the segment they need either to be deleted or to be defined using the element editor. The periodicity field stores a number of repetitions of the segment, for example used to calculate tunes etc.

"Invert dipole polarities" inverts the polarity of all dipoles and combined function magnets in the lattice.

"Set all element apertures" overwrites all element apertures with the values given in the input fields. Warning: all individual aperture settings, if any, will be lost!

The text editor shows the file exactly as it will be saved to disk, see sec.3.1 for explanation. The "test" button performs a syntax check.

2.4 Design: optics

The lattice structure appears on the optics panel, and a menu to select constraints pops up:

- "periodic" will try to find a solution with same parameters at both ends
- "symmetric" will try to find solution with $\alpha_x = \alpha_y = \eta' = 0$ at both ends, so appending the mirror image makes it periodic.
- the "single pass" options calculate the optics starting at its left or right end or from one of the "optics markers" if any.

If periodic or symmetric fails, the single pass solution from initial (left) values is calculated.

If the periodic solution was found, the tune diagram pops up (see below).

Beta functions and dispersion will be shown (β_x in blue, β_y in red, η in green). The table at right gives in its upper part total values of the lattice (some of them only valid, of course, if the lattice would be repeated periodically). In the lower part, local optics values are displayed. The location can be selected by right-clicking on the plot.

Moving the mouse over one of the elements displayed at the bottom of the figure shows its name. Left-click on an element and releasing the mouse button after moving to one of the "knob" fields, connects the most important parameter of this element to the knob. Variation by moving the slider or entering numbers changes the optics. The range of variation is given by the fields at the bottom of the knob, which are adjusted automatically, but can be changed using the "<>" and "><" buttons.

Left double-click on an element opens the panel from the element editor which allows to change all parameters.

Buttons at the top of the plot allow to zoom in: calculation through elements proceeds in slices of length corresponding to 1 pixel on screen.
The button field at lower left:

"Start" again selects the constraints.

"Envelopes" is for showing the beam size: if the equilibrium values of the lattice are used, an emittance coupling factor has to be given, since there is virtually no vertical emittance in an ideal flat lattice. The envelope plot will show the 1-sigma beam size and the element apertures. Dispersion is included in the horizontal beam size, its contribution shown as a dashed line.

"Momentum" displays the optics for $\pm\Delta p/p$ and on-momentum. One may select to include sextupoles or not. Click Momentum again to switch off this mode. Note, that a sextupole connected to one of the knobs of course can only affect the off-momentum curves.

"Tune Matrix" gathers quadrupoles in two groups depending on the sign of the k-value and establishes a sensitivity matrix for a relative change of strength. This allows relatively smooth tuning of the lattice within a small range.

"Matching" opens the panel for iteratively adjusting beam parameters. Matching can be done from any location to another in the lattice, i.e. begin, end or optics markers. Constraints at an intermediate marker may be added.
On the left panel, select parameters to adjust and enter target values. On the right panel, select elements to be used for matching. If the number of elements is equal or larger than the number of constraints, the "go" button is enabled and matching may start.
The algorithm is a Newton search using the inverse square sensitivity matrix of the most effective knobs as tangent for extrapolation (see K.Wille's book). This method converges quadratically but is rather fragile. Reducing the "fraction" of iteration to be applied stabilizes. Other parameters are number of iterations and required precision for termination.
If the iteration was successful, one may watch the solution ("show"), "accept" or "reset" it to the initial. "retry" returns to the first screen to change the settings and try again.
Known bugs: Symmetric or periodic option does not yet work well. Previously selected intermediate constraints are not reset on restart.

"write OMK" writes the current optics into the optics markers elements, which can be selected on a panel that will pop up.

"Print" prints the plot stretching it to the available area and refining it to the available resolution of the selected printer.

"→Plot" writes a graphics file `(nam)_beta.wmf` with betafunctions, resp. `(name)_envel.wmf` if envelope mode was selected.

"→txt" writes a text file `(nam)_beta.txt` of plotted data (for postprocessing using other tools).

"→GP" writes text files `(nam)_beta.out` with the beta functions and `(nam)_mcod.out` with element data to be read by a GNU-plot command file `(nam)_beta.gp`, which is also created. Running GNU-plot on this file (outside OPA) then will create an EPS file `(nam)_beta.eps`.

"Exit" terminates the optics design. Note that the question to save the data refers only to an internal save to further proceed inside OPA, but does not save the data to the file!
If optics design ends with a periodic solution, the options "sextupoles" and "tracking" become enabled.

There is an embarrassing bug that could not yet be fixed: if the optics plot window is covered up many times by other panels, Windows runs out of memory probably due to some OPA-bug in repainting the plot window, and every user action will repeat this error. In the worst case, OPA has to be killed using the Windows tasks manager.

2.5 Tune diagram

The tune diagram pops up when a periodic solution is found, it shows the working point at the centre and the resonance lines in the neighbourhood.

The lines $aQ_x + bQ_y = c$ to be shown are selected by the order buttons and the checkboxes:

- Order buttons select $|a| + |b| \leq$ order.
- "nsys" not checked selects only systematic lines, where $c \bmod P = 0$, with P the periodicity of the lattice.
- "skew" not checked selects only regular lines, where $b \bmod 2 = 0$.

Buttons at bottom modify the plot range.

"Export" writes a graphics file `(nam)_tuneplot.wmf`.

Double-clicking the image saves the bitmap to the clipboard (this works for all plots in OPA except the "Optics Design" plot).

2.6 Design: sextupoles

Sextupoles, octupoles and decapoles may be used to optimize the dynamic aperture after chromaticity correction.

The modes of the sextupole Hamiltonian are calculated in first and second order, (see section 4.1): there are 10 terms in first order (2 linear chromaticities and 8 resonances) and 13 in second (2 quadratic chromaticities, 3 amplitude dependant tune shifts (ADTS) and 8 octupolar resonances). The first order modes of the octupole Hamiltonian are calculated and added to the 13 second order sextupole modes. Also cubic chromaticities are calculated. Chromaticity and path length calculation is done by numeric differentiation of the dispersive orbit, whereas all other quantities are calculate analytically.

All Hamiltonian modes are normalized to betatron amplitudes of $2J = 1$ and a relative energy deviation of $\Delta p/p = 1$. ADTS values are displayed as $\partial Q/\partial(2J)$.

As much as possible is calculated only once at start of the sextupole module. This may cause some delay depending on the total number of sextupole kicks in the lattice (which depends on the number of slices per sextupole etc.). But afterwards the iterations will be fast.

A tune diagram will also pop up. In addition to the tune diagram opening with linear optics, it shows the expected tune spread of the beam: the magenta/cyan parabola shows the tune variation due to chromaticity and the green straight lines due to betatron amplitudes.

The sextupole panel lists at left the 25 Hamiltonian modes plus a sum of total sextupole strength. The numbers are the absolute value of the (complex) mode and given in SI-units. To each resonant mode exists a complex conjugate, which is not shown.

The "periods" field will be set to the number of periods of the lattice, but may be changed to study how the resonant terms vary due to interference. For the non-resonant terms, the period number is just a scalar multiplication factor. A value of 0 means ∞ and is applied only to the resonant terms, of course.

The bars are for comparison and visualize the contribution to the minimizer's penalty function, they are products of the modes' absolute values h_{jklmp} with relevant amplitudes $(2J_x)$, $(2J_y)$ and δ, and weighting factors: the relevant betatron amplitudes and momentum range are given by the fields at bottom, practical units here are mm·mrad and %. Another (empirical) factor applies to the resonant terms to make them comparable, otherwise they would be invisible compared to the chromaticities. The weighting factors are given by $(2^w - 1)$ with w stepped up/down by the " + " and " − " buttons. For non-resonant, additive terms like chromaticities and tune shifts, a target value can be set. The check boxes in the "inc" row include/exclude the related quantities to the minizer.

The penalty function is calculated as

$$\sum_{jklmp} \left((2^w - 1)\, 10^{W_R}\, (2J_x)^{(j+k)/2}\, (2J_y)^{(l+m)/2}\, \delta^p\, |h_{jklmp} - t_{jklmp}| \right)^2$$

For the non-resonant mode, where $\Im(h) = 0$, a target value t_{jklmp} can be given, which is always zero for the resonant modes. For the resonant modes, a general weighting factor W_R is given in the fifth field at the bottom of the panel, to increase their weight, because due to interference they are smaller numbers than the additive, non-resonant terms, but due to resonant amplification they may be more harmful.

The right side of the panel shows the integrated strengthes of the related sextupole resp. combined function magnet families. Maximum strength and step size for pressing the " > " and " < " buttons are given below. " >> " and " << " buttons apply 20 steps. "off" sets to zero, "res" restores the initial value. The "lock" check box excludes the family from the minimizer. Underneath are windows to set maximum strength and step size.

For automatic adjustment of linear chromaticities to the target values, two sextupole families have to be selected by clicking the check boxes in the row labeled "ξ". If these sextupoles have no dispersion, chromaticity correction will be impossible and an error message is shown. There may be a small deviation from the target value of chromaticity. This is due to the fact, that linear, quadratic and cubic chromaticities are obtained from numeric differentiation, whereas the chromatic sextupole values are calculated using the simple 2×2 matrix containing the well known sums over beta functions and dispersion.

The "select" button launches a window for visualization of the first order resonances, it displays the sextupole kick vectors in the complex plane. Pressing "select" again toggles between the 8 resonance modes which will be highlighted at left. The sum is shown as circle. An 8^{th} plot shows the sum vectors of the 8 resonance modes for comparison.

If there are octupoles in the lattice, they will be shown underneath the sextupoles, and a row of checkboxes labeled "O" appears besides the 2nd order chromaticities and amplitude dependant tune shifts. Checking these boxes activates a singular value decomposition (SVD) routine [7] which is controlled by the buttons appearing underneath the octupoles: The "Condition" and "Nweight" labels shows the ratio of smallest non-zero to largest value of the weight vector and the number of non-zero values. The $-/+$ buttons may be used to filter these values and improve the condition. The "SVD" button performs a calculation which can be canceled using the "undo" button. Checking the "auto" box enables automatic SVD after each change of a sextupole. SVD uses all available octupoles, but checking the "lock" box at an octupole excludes it.

If there are decapoles in the lattice, they will be shown underneath. They only affect the cubic chromaticites and may be modified manually only.

A cubical fit for the momentum dependant orbit length (longitudinal chromaticity) is given at the bottom, numbers given are the first three orders of the momentum compaction factor multiplied by the lattice length.

"start" starts the minimizer, which uses the Powell algorithm [7]. The initial step size for sextupole variation may be set. The minimizer will inform on its progress by listing the current value of the penalty function (normalized to its start value) and also by writing messages to the message window on the OPA main panel. After successful termination, the penalty function is shown in blue, after interrupt in magenta.

The minimizer does not use the octupoles, however if the "auto" option for SVD is activated, the octupoles are set after each minimizer step. In the same way the chromaticity sextupoles are set.

"Exit" terminates the sextupole programs and asks if the new sextupole values should be saved. Note that the question to save the data refers only to an internal save to further proceed inside OPA, but does not save the data to the file.

2.7 Tracking: phase space

This panel shows Poincaré plots of particle motion in horizontal and vertical phase spaces (x, x') and (y, y') (*it is* x', y' *although the axis are labeled* p_x, p_y). The circles or ellipses show the linear acceptance given by the physical aperture. When starting a particle, the effective apertures are also shown, which are reduced due to coupling assuming elliptical beam pipes (see sec.4.5).

By left-clicking in the diagrams or by entering numbers in the centre panel, the starting conditions are selected. A momentum deviation may be entered in the "dp/p" field. "run" tracks the particle a number of turns as given in the top panel, "more" adds more turns, "clear" clears the panel, "exit" terminates the program.

A Fourier spectrum of particle motion is calculated after tracking and shown in the lower diagrams. The algorithm used is FFT with sine window and $\sin x/x$ peak interpolation to obtain frequency and amplitude of the peak in order to identify (guess) the underlying resonance [5]. Results are shown in the lower panel. The tune considered as the fundamental is also marked in the tune diagram. The "param" button opens a panel to set parameters for resonance guessing: "fundamental tune range" sets the maximum deviation of the guessed tune from the linear tune. "peak identification tolerance" sets a parameter p defining $\Delta Q = p/c$ as the maximum tune deviation, with c the order of the resonance. The two filter levels are for accepting peaks and for the range of the plot.

By default, tracking uses the apertures of the elements to test on particle loss. By unchecking "element apertures", the apertures in the "Ax", "Ay" fields are used instead. Aperture check assumes an elliptical beam pipe and tests for $(x/a_x)^2 + (y/a_y)^2 \leq 1$. Note, that the aperture check may underestimate losses, because internally, sequences of linear elements are concatenated into matrices at start. Aperture checks are only done at locations of nonlinear kicks and at the track point. Therefore, particle tracking may also paint outside the linear acceptance ellipse.

The trackpoint by default is at the begin/end of the lattice, but the field "trackpoint" allows to select any location. The ellipses indicating the physical acceptance will change accordingly.

The "amplitude dependent tune shifts" panel performs three series of trackings stepping up the initial angles x', y' in steps as given in the "steps" field until the maximum betatron amplitudes given by physical acceptance, are reached. The first (third) series goes in horizontal (vertical) direction up to the maximum betatron amplitude with a tiny vertical (horizontal) amplitude in order to excite coupling. The second series sets the initial coordinates for a constant coupling. Coupling and maximum amplitudes are defined in sec.4.5.

The identified tunes are marked during execution in the tune diagram, and afterwards they are plotted vs. betatron amplitude, or vs. "ping", i.e. the initial kick angle. The three series

correspond to the lines blue-X, lilac-X (both showing $\Delta\nu_x$ vs. $2J_x$) and purple-Y (showing $\Delta\nu_x$ vs. $2J_y$) in the left diagram, and to the lines lilac-X (showing $\Delta\nu_y$ vs. $2J_x$), purple-Y and red-Y (both showing $\Delta\nu_y$ vs. $2J_y$) in the right diagram.

Since the amplitude is derived from the height of the fundamental peak in the Fourier spectrum, it may happen in case of very strong non-linearity, that the curve "returns" towards the origin, if a harmonic peak grows on expense of the fundamental peak. It may also happen, that the tune gets stuck on a resonance (i.e. the particle is trapped in an island in phase space) and the tune shift vs. amplitude shows a flat line.

If theoretical amplitude dependent tune shifts have been calculated previously in the sextupole module, they are shown as dotted line for comparison.

2.8 Tracking: momentum

Actually, this module does no tracking but calculates the linear optics along the off-momentum orbit. It works for periodic and also for single pass systems: the checkbox at top right makes the selection and is checked at start if a periodic solution was previously calculated. The range of momentum variation and the number of steps may be set, and if a periodic or single pass solution is requested (the corresponding checkbox is set following previous "Optics Design" calculations). "Go" starts the calculation. A tune diagram pops up in case of periodic solution.

Afterwards, several plots may be selected by the radio buttons at left. Each curve is fitted by a polynomial, the order to be selected by the field at left, where also the coefficients will be shown in a table. The "Units" button switches between plot units and SI units for displaying the coefficients.

In the tune plots (first four radio buttons), also the theoretical values (actually calculated in the same way by numeric differentiation) are show by solid lines, if the sextupole module was used before. The chromatic beam footprint will be shown in the tune diagram.

"→WMF", resp. "→TXT" writes a graphics file, resp. a text file of the data named `(name)_momentum_N.wmf`, resp. `.txt`, where `N` is the number of the plot (radio button). Double-clicking the image copies it to the clipboard.

Up to four parameters may be selected for automatic minimization by checking the box after the name of the parameter. It then appears as a button label in the minimizer panel at bottom. Clicking this button shows the parameter in the plot and allows to edit target values as polynomial coefficients in the fit panel at right, which are shown in pink in the plot. The minimizer is of Powell type [7] and works on the penalty function

$$\sum_f \sum_\delta w_f \left(f(\delta) - f_T(\delta)\right)^2,$$

with δ the array of momentum values covering the selected range, f the momentum dependent parameter, f_T the target for this parameter, and w_f a weighting factor. The latter is entered left of the button. Only nonlinear elements are available for minimization, the minimizer knobs are

the strengths normalized to the initial values. "Optimize" starts the minimization, "Break" interrupts it, "Reset" restores the initial values. "Plot absolute/relative" toggles between showing parameter and target, or the difference between both.

Target values are saved on exit, however this has been implemented yet only for tunes and path length up to 12th order.

2.9 Tracking: dynamic aperture

There are three modes: x vs. y, x vs. $\Delta p/p$ and y vs. $\Delta p/p$. When selecting the first, a $\Delta p/p$ offset may be given, for the others it is a $\Delta p/p$ range. The plot will show the geometric acceptance based on linear beam dynamics and the grid for probing stability by starting particles at these locations. Grid parameters may be stepped up and down by the "cells" buttons. The geometric acceptance is modified by setting the apertures as described in sec.2.7. Other parameters are the number of turns (*and the location of the trackpoint*).

"Start" starts probing the grid: surviving particles are shown in green, lost ones in red. "Export" writes graphic files `(name)_dynap_xy/_xp/_yp.wmf`. Double-clicking the image copies it to the clipboard.

In x vs. y mode, the geometric acceptance is shown as a blue polygon. It is given by the common area of the projections of all the elliptical apertures to the track point (see appendix 4.5). This geometric acceptance includes non-linear and chromatic contributions to orbit and beta functions but else assumes linear transformations. In tracking, it may happen, that particles outside the geometric acceptance survive for two reasons: 1) aperture checks are only performed at non-linear elements (because to speed up tracking series of linear elements are concatenated and stored as matrices), and 2) the non-linear eigenfigure of betatron motion may be different from an ellipse.

2.10 Tracking: Touschek lifetime

This module for Touschek lifetime calculation proceeds in two steps:

1. Work sheet to calculate lifetime related parameters in a linear model: one may enter parameters in the left panel and see the results in the right panel and in the plots. Available plots include

- the beta functions,
- the lattice invariant $\mathcal{H}$,
- the rms beam envelopes,
- the apertures,
- the bunch volume,

- the momentum acceptances from RF (green) and from apertures (brown),
- the ζ-parameter for the Touschek function (see section 4.2)
- the local loss rate in linear scale, and
- the local loss rate in logarithmic scale.

2. Tracking for local dynamic momentum acceptance: after pressing "track", the program will step along the lattice in steps of Δs and start particles on-axis but with momentum deviation $\Delta p/p$ to simulate Touschek scattering. A binary search on $\Delta p/p$ determines the minimum and maximum values of $\Delta p/p$ accepted at the particular location. Only locations where $\mathcal{H}$ (and its nonlinear equivalent) has changed need to be tested, therefore the loop jumps over non-dipole elements and just copies the previous momentum acceptance data.
During execution, the plot will switch to momentum acceptance to show the progress of the calculation. "Break" allows to interrupt. When done, all calculations for ζ, loss rates and lifetime like in the linear case are done and the results are added to the plots. In the ζ plot, 2 additional lines for positive (red) and negative (blue) dynamic momentum acceptance will appear. In the loss rate plot it is only one line for the total losses (pos. and neg.).

Input parameters:

- Energy may be changed for testing, but the original value will be restored on exit.
- Coupling is the emittance ratio.
- Total beam current in the machine, and
- number of bunches give the charge per bunch.
- The cavity voltage defines bunch length and RF momentum acceptance. If it is lower than the energy loss per turn, the bunch length is set to the ring circumference and the momentum acceptance is zero.
- The harmonic number contains RF wavelength. The number of buckets can not be higher than the harmonic number, this is checked.
- The longitudinal resolution defines Δs for tracking along the lattice.
- Momentum resolution defines the precision of the binary search.
- Numbers of turns to track to decide on loss or survival.
- Residual gas is defined by atomic number, atoms per molecule and partial pressure.

The gray fields show quantities derived from the input values, which can not be modified.

"Track" starts the tracking, which is a 4-D tracking, i.e. keeping the momentum constant.

"Track sync" is an improvised 5-D tracking, varying the momentum deviation $\delta = \Delta p/p$ in turn k simply as $\delta_k = \delta_o \cos(2\pi\nu_s k)$, with ν_s the synchrotron tune. The implementation is very inefficient, and therefore this tracking is rather slow.

"Export plot" writes a file `(name)_touschek_N.wmf`, where `N =0..8` is the number of the plot (`N=0` for beta function plot etc.).
"Export data" writes a file `(name)_touschek.txt` with all plotted data.
"ZAPLAT" writes a file `(name)_zaplat.dat` to be used as input file `zaplat.dat` by the ZAP code [2] for calculation of intra beam scattering. Note, that aperture data as needed for Touschek lifetime calculations by ZAP are not included.

2.11 Extra: Geometry

A plot of the lattice layout: press and hold the left mouse button to mark a rectangle to zoom in. The curved arrow buttons on top will follow the lattice back or forth, the third button zooms out again.

If there are marker elements named "`center`" in the lattice, the plot will be centered at the center of gravity of these markers.

Unchecking the "fix aspect ratio" box stretches the plot horizontally and vertically to the canvas, otherwise (default) same scales are used horizontally and vertically.

Parameters in the table control width (and length of some) elements.

"wmf" writes a file `(file)_geometry.wmf`, "data" writes three files, named `markers.txt`, `devices.txt` and `orbit.txt` containing data of points where a new straight or curved section starts, the data of the ploygons relative to the markers and the data of the orbit. Markers and orbit can be visualized by checking the corresponding boxes.

2.12 Extra: Orbit

If monitors, horizontal and vertical correctors have the special names `MON`, `CH` and `CV`, the family is expanded in single elements names `MONnnn` etc., where `nnn` is a number. Only these elements will be used for orbit correction. Correctors or monitors with other names remain families and can only be used manually. Monitors and correctors will show on the Orbit panels as green, blue and red objects. Correctors can be dragged to knobs to use them manually, a monitor can be dragged to the BPM-field. It will show its reading there. By switching from "actual" to "Reference" a target value can be entered for x and y and is written to the BPM after pressing "set".

The "status + statistics" panel at lower left allows to set the starting point and to include or exclude nonlinear elements. The statistics panel shows mean, rms and max values for orbit relative to reference ("BPM"), for absolute orbit at all elements ("all") or for the corrector kicks ("Corr") – press to toggle.

The "plot" panel at lower right allows to choose between corrector and orbit+BPM plots. If "keep max." is checked, no autoscaling of plot is done.

The "misalign and correct" panel sets misalignments and performs orbit corrections: Misalign-

ments are entered in micron and will be applied to all magnets (not the correctors) as Gaussian distributed random numbers with the given cut-off in sigma. "seed" determines the series of random numbers. "Set" applies the misalignments, "zero" sets them to zero, and "re-set" applies a previous setting again (to save retyping the input fields).

"Corr" calculates the response matrix, sets-up the SVD and performs orbit correction. The results as shown below the "Corr"-button are:
"COD": no correction done,
"zeroed": success: orbit agrees with reference,
"minimized": success: orbit does not agree with reference, but does not converge any further. This is typically the case if there are less correctors than monitors.
"no convergence": failure: too many iterations, the orbit loop did not converge.
"failed": complete failure, beam ran off.

The buttons "Corr=0" and "BPM=0" set all correctors, resp. all BPM references to zero.

The figure displays the SVD weight factors, where the excluded (zero) values are shown darker. The small button toggles "X" and "Y", and the slider allows to reduce the weighting factors.

OPA may crash and get stuck on orbit correction, if the orbit is very sick; catch still missing.
In case of lattices without correctors or BPMs, orbit correction can be started anyway and produces an error; catch missing.
Orbit corrections works yet only for periodic solution, implementation for single pass would be straightforward but is not yet done.

2.13 Extra: Currents

If allocation and calibration files were given in the lattice file (see sec.3.1), the magnet currents are calculated and shown. "Power supply" is the name of a magnet family, "N" is the number of magnets in this family, and "Current" is the current, which is the average of the current calculated for the single family members (differences may be due to different magnet types connected in a family). Clicking a field shows at right the family member data.

"Snap export" writes `(file).snap` for uploading to EPICS control system [1].

3 Data

3.1 The input/output files

Files with extension `.opa` are read and written by OPA. The syntax is simple (look at an example file!):

[1] This is for internal use at SLS and probably would have to be modified for other machines.

First are the global parameters like beam energy etc, →3.2, then comes the list of elements, →3.3, and and finally the list of segments, →3.4, which will be unpacked recursively to generate the lattice.

All inputs are optional, since OPA may start without lattice as well. For parameters of an element not given in the input file, OPA assumes reasonable defaults. If a parameter is mistyped, OPA will ignore it and set it to the default value.

Text in curly brackets {...} is treated as comment, i.e. ignored by OPA, except text bracketed by {`com ... com`}, which is the "official" comment text like lattice title and some notes. Further comments visible in the `*.opa`-file are generated by OPA for the user's convenience. Comments written manually into the `*.opa` file will be lost.

To use the option of exporting magnet currents and channel names (→2.13), a link to the corresponding allocation and calibration files is required and should be given at the beginning of the `*.opa` file:

```
allocation = (allocation-file).dat; calibration = (calibration-file).dat;
```

3.2 Global Parameters

This is only the comment {`com ... com`}, the beam `energy`, optionally the allocation/calibration links, *and optionally, explicit initial parameters (to be removed).*

3.3 Elements

Description of elements, meaning of the parameters and how they appear in the input file `(examples)`. All parameters can be seen in the OPA Editor.

All elements have a length L [m] (=0 in some cases) and horizontal and vertical apertures A_x, A_y [mm] (half apertures, internally assumed rectangular not elliptic).

- Driftspace:
  ```
  D2 :  Drift, L = 0.200000, Ax = 35.00, Ay = 17.00;
  ```
- Quadrupole: `K` is strength $b_2 = B/(B\rho)$ [m^{-2}] , `K`> 0 horizontally focusing.
  ```
  Q1 :  Quadrupole, L = 0.200000, K = 4.350000;
  ```
- Bending magnet: angle `T` [°], in/out edge angles `T1/T2` [°] and corresponding fringe field parameters `K1IN/EX, K2IN/EX` [8], gap `GAP` [mm] and gradient `K` (like quadrupole). Angle can be positive or negative depending on bending direction. An edge angle of same sign like the bend angle makes the bend vertically more focusing, i.e. a rectangular bend always has `T1=T2=T/2` no matter if `T` is pos. or neg.
  ```
  B1 :  Bending, L = 1.000000, T = 45.00000, K = 0.000000,
  T1 = 22.50000, T2 = 22.50000, Gap = 0.00,
  K1IN = 0.0000, K1EX = 0.0000, K2IN = 0.0000, K2EX = 0.0000;
  ```

- Sextupole : if $L > 0$, `K` is the sextupole strength b_3 [m^{-3}], if $L = 0$, `K` is the integrated sextupole strength $b_3 L$ [m^{-2}]. `K` > 0 is horizontally focusing for $x > 0$.
  Sextupole strength is defined as $b_3 = B''/(B\rho)/2 = B_{\text{poletip}}/R^2/(B\rho)$.
  `N` is only meaningful for $L > 0$, since the sextupole is modeled as
  $N\times$ [driftspace $L/(2N)$ - thin sextupole kick $b_3 L/N$ - driftspace $L/(2N)$]
  `SD : Sextupole, L = 0.200000, K = -10.000000, N = 4;`
- Combined function magnet: has the same parameters like the bending magnet except `K2` fringe field parameters. In addition it also may contain a sextupole strength `M`$= b_3$ [m^{-3}] and the corresponding number of subdivisions, `N`.
  Bend angle `T=0` is allowed and describes a quadrupole/sextupole combination:
  `Q3C : Combined, L = 0.400000, T = 0.00000, K = 4.350000,`
  `M = 12.000000, T1 = 0.00000, T2 = 0.00000, K1IN = 0.0000,`
  `K1EX = 0.0000, Gap = 0.00, N = 5;`
- Undulator: period `Lamb` [m], peak field `Bmax` [T] – Note: since the peak field is explicitly given, the undulator is the only element where the optics changes with beam energy, whereas all other element are described energy indepently! - filling factors `F1,2,3` ($\rightarrow$4.3) and `Gap` [mm]. Internally, the undulator is represented by a series of rectangular dipoles, where the two end poles have $1/4$, $-3/4$ of peak field to center the trajectory. Only planar undulators with vertical field are possible!
  `U19 : Undulator, L = 0.912000, Lamb = 0.019000, Bmax = 1.000000,`
  `F1 = 0.636620, F2 = 0.500000, F3 = 0.424413, GAP = 5.000,`
  `Ax = 50.00, Ay = 2.00;`
- Marker: no parameters.
- Optics Marker: useful to store beam parameters and for matching from/to.
  `OM1 : OpticsMarker, Ax = 35.00, Ay = 17.00, BetaX = 7.657791,`
  `AlphaX = 0.000000, BetaY = 0.723110, AlphaY = 0.000000,`
  `EtaX = -0.123958, EtaXP = 0.000000, EtaY = 0.000000,`
  `EtaYP = 0.000000, OrbitX = 0.000, OrbitXP = 0.000,`
  `OrbitY = 0.000, OrbitYP = 0.000, OrbitDPP = 0.000;`
- Solenoid: `K` is solenoid strength $(B_s/(2B\rho))^2$.
  incomplete: beam rotation, contribution to 2Q resonances, to path length and to radiation integrals are not included: better don't use it.
  `SO1 : Solenoid, L = 0.200000, K = 0.50000;`
- H-corrector `DXP` is horizontal kick in mrad, however it has only effect in the orbit module, is ignored elsewhere. If the name is `CH` it will be used for orbit correction.
- V-corrector `DYP` is vertical kick. If the name is `CV` it will be used for orbit correction.
- Monitor, no parameters. If the name is `MON` it will be used for orbit correction.
- Other elements are infrequently used and may be seen in the OPA editor.

3.4 Segments

A segment is a line-up of elements and segments. Many segments may be defined, the segment to be used to built the lattice is selected interactively later.

A minus sign reverses the segment. A factor repeats it. Correct direction of bendings with different edges is taken care off internally.

In the example below, underlined names are segments, the others are elements. Segment hierarchy requires, that sub-segments to be used by a segment have to appear earlier in the file, otherwise it is an error. This also avoids circular references.

```
MS :  DS1, SSA, DSX, QSE, DS2, QSF, DSX, SSB, DS3, QSG, DS;
TMS :  TS, MS;
TEST:  -TML, TMS, 3*HUGO, QF, 5*XYZ;
```

The field `nper=n` appearing somewhere in a segment line-up defines the periodicity, i.e. the factor `n` is applied for convenience to tunes, sextupole terms etc of the lattice buit from this segment, e.g.

```
ARC :  HALF, XXX, -HALF, nper=6;
```

Note: if one likes to split a bend, for example to put a marker in the centre, and a non-zero gap is given, then one has to set `K1EX=0.0` otherwise the internal edge where the dipole is split, will have an effect on the vertical focusing! Example:

```
BHALF: Bending, L=..., T=..., T1=..., T2= 0.0, Gap=..., K1IN=..., K1EX=0.0;
BEND: BHALF, -BHALF;
```

4 Theory

4.1 The sextupole Hamiltonian

Calculations of 1st and 2nd order sextupole terms have been done by Johan Bengtsson [5, 6]. We add here the multiplication factors for N periods:

For first order, the complex factor a_N to transform the Hamiltonian mode for 1 period into N periods, i.e. $h^N = a_N \cdot h$ is given by [5]

$$a^N_{\vec{m}} = \sum_{n=0}^{N-1} e^{in\vec{m}\vec{\mu}} = \frac{1 - e^{iN\vec{m}\vec{\mu}}}{1 - e^{i\vec{m}\vec{\mu}}},$$

with $\vec{m} = (j-k, l-m)$ the mode of h_{ijkl} and $\mu = 2\pi(Q_x, Q_y)$ the ring tune. Defining $\psi = \vec{m}\vec{\mu}/2$ this can be expressed as

$$a^N_{\vec{m}} = \frac{\sin\psi + \sin(2N-1)\psi}{2\sin\psi} + i\frac{\cos\psi - \cos(2N-1)\psi}{2\sin\psi}$$

In the limit $N \to \infty$ we may neglect the fast varying term and get

$$a^{\infty}_{\vec{m}} = \frac{1}{2} + i\frac{1}{2}\cot\psi \qquad \longrightarrow \qquad |a^{\infty}_{\vec{m}}| = \frac{1}{2\sin\psi}$$

revealing the resonance denominator.

For the 2nd order we define

$$\sum_{\ominus}; \quad \sum_{\oplus}; \quad \sum_{\odot} = \sum_{s=1}^{S}\sum_{s'=1}^{s-1}; \quad \sum_{s=1}^{S}\sum_{s'=s+1}^{S}; \quad \sum_{s=1}^{S}\sum_{s'=1}^{S} r_s r_{s'} e^{-i(\vec{m}\vec{\phi}_s + \vec{m}'\vec{\phi}_{s'})}$$

with $r_s = (b_3 L)_s \beta_{xs}^{\frac{(j+k)}{2}} \beta_{ys}^{\frac{(l+m)}{2}}$ and $\vec{\phi}_s = (\phi_{xs}, \phi_{ys})$ containing beta functions and phases of the sextupole s.

The 2nd order sextupole cross talk terms contributing to the octupolar resonance $\vec{m} + \vec{m}'$ are given for one period by

$$h_{\vec{m}\vec{m}'} = iA\left(\sum_{\oplus} - \sum_{\ominus}\right)$$

with A a scalar factor[6]. For N periods this becomes

$$h^{N}_{\vec{m}\vec{m}'} = iA\left(a^{N}_{-(\vec{m}+\vec{m}')}\left(\sum_{\oplus} - \sum_{\ominus}\right) + b^{N}_{\vec{m}\vec{m}'}\sum_{\odot}\right)$$

with a^N as defined above (inserting $-(\vec{m}+\vec{m}')$ for $\vec{m}$) and

$$b^{N}_{\vec{m}\vec{m}'} = \frac{-\mathcal{S}(N,\vec{m},\vec{m}') + \mathcal{S}(N,\vec{m}',\vec{m}) - i\cdot(-\mathcal{C}(N,\vec{m},\vec{m}') + \mathcal{C}(N,\vec{m}',\vec{m}))}{8\sin\psi\,\sin\psi'\,\sin(\psi+\psi')},$$

$$\mathcal{S}(N,\vec{p},\vec{q}) := \sin\vec{p}\vec{\mu} - \sin(-N\vec{p})\vec{\mu} + \sin((1-N)\vec{p}+\vec{q})\vec{\mu} - \sin((1-N)\vec{p}-N\vec{q})\vec{\mu}$$

and $\mathcal{C}$ the same with cos instead of sin.

For $N \to \infty$ we again neglect the fast varying terms and get

$$b^{\infty}_{\vec{m}\vec{m}'} = \frac{-\sin 2\psi + \sin 2\psi' - i\cdot(-\cos 2\psi + \cos 2\psi')}{8\sin\psi\,\sin\psi'\,\sin(\psi+\psi')} \to |b^{\infty}_{\vec{m}\vec{m}'}| = \frac{1}{4\sin\psi\sin\psi'}\frac{\sin(\psi-\psi')}{\sin(\psi+\psi')}$$

4.2 Touschek lifetime

Touschek lifetime is given by [1] [2] [3]

$$\frac{1}{T} = \frac{r_e^2 cq}{8\pi e\gamma^3\sigma_s}\cdot\frac{1}{C}\oint_C \frac{F(\zeta(s))}{\sigma_x(s)\sigma_y(s)\sigma_{x'o}(s)(\delta_{\mathrm{acc}}(s))^2}\,ds \qquad \text{with} \;\; \zeta(s) := \left(\frac{\delta_{\mathrm{acc}}(s)}{\gamma\sigma_{x'o}(s)}\right)^2.$$

r_e is the classical electron radius, q the bunch charge, σ_s the bunch length (assumed to be constant along the lattice), C the machine circumference, σ_x and σ_y the transverse rms beam envelopes and $\sigma_{x'o}$ the divergence for $x \approx 0$, given by

$$\sigma_{x'o}(s) = \frac{\epsilon_x}{\sigma_x(s)} \sqrt{1 + \frac{\mathcal{H}(s)\sigma_\delta^2}{\epsilon_x}},$$

with σ_δ the rms energy spread and $\mathcal{H}$ the lattice invariant.

$\delta_{\rm acc}(s)$ is the local lattice momentum acceptance, which is the minor of the lattice and the RF acceptance. In the linear case, the lattice momentum acceptance at $s = s_o$ is given by

$$\delta_{\rm acc}^L(s_o) = \min \left(\frac{a_x(s)}{\sqrt{\mathcal{H}_o \beta_x(s)} + \eta(s)} \right)$$

where a_x is the horizontal aperture. In the non-linear case it is obtained from tracking.

The "Touschek function" $F(\zeta)$ is defined as

$$F(\zeta) = \int_o^1 \left(\frac{1}{u} + \frac{\ln u}{2} - 1 \right) e^{-\zeta/u} \, du$$

For small arguments $\zeta < \zeta_{\rm small} = 0.0013$ the asymptotic expression $F(\zeta) = \ln(E/\zeta) - 3/2$ is used, with $E = 0.5772$ Euler's number.

For large arguments, F becomes very small and is dominated by rounding errors, so for $\zeta > \zeta_{\rm big} = 22.8$ it is set to $F(\zeta_{\rm big}) = 10^{-16}$ (usually lattice locations where this happens are irrelevant anyway for the final lifetime result).

For $\zeta_{\rm small} < \zeta < 10$ a fair approximation is given by a 7th-order polynomial

$$F(\zeta) \approx \exp \sum_{k=0}^{7} A_k (\ln \zeta)^k, \quad \text{with} \quad \begin{aligned} A = \{ \quad & -3.10811, -2.19156, -0.615641, \\ & -0.160444, -0.0460054, -0.0105172, \\ & -1.31192 \cdot 10^{-3}, -6.3898 \cdot 10^{-5} \quad \} \end{aligned}$$

The agreement within $\pm 2.5\%$ compared to the integration is acceptable considering that the integral itself deviates up to 10% from detailed Monte Carlo simulations of Touschek scattering [4].

4.3 A simple undulator model

The alternating field of an N-pole wiggler (or undulator) can be well approximated by a series of $2N$ rectangular dipole magnets, where the end poles on both sides are attenuated. The field of half pole k is $B_k = p_k \hat{B}$ with $\hat{B}$ the maximum field occurring in the central poles only, and

$$p_k = \left\{ \frac{1}{4}, -\frac{3}{4}, 1, -1 \ldots 1, -1, \frac{3}{4}, -\frac{1}{4} \right\} \quad k = 1, 2 \ldots 2N$$

for optimum centering of the wiggling motion of the electron beam.

A general filling factor, i.e. the ratio of rectangular bend length to half pole length ($= \lambda/2$) is defined by

$$f_n = \frac{1}{\lambda} \int_o^\lambda \left| \frac{B_y(s)}{\hat{B}} \right|^n ds$$

where f_1 affects the orbit, and f_2, f_3 damping partitions, emittance and energy spread. An ideal sinusoidal wiggler has

$$f_1 = \frac{2}{\pi} = 0.637 \qquad f_2 = \frac{1}{2} = 0.5000 \qquad f_3 = \frac{4}{3\pi} = 0.424.$$

Taking into account that the variations of beam functions within a half pole of a wiggler are rather small, defining $\hat{h} = \hat{B}/(B\rho)$ the maximum curvature, and further assuming that a wiggler usually has no gradient (would affect I_4 only), the contributions of half pole k to the synchrotron radiation integrals [9] are approximately given by

$$\Delta I_{1k} = \int_0^{\lambda/2} \frac{\eta}{\rho}\, ds \approx f_1 \frac{\lambda}{2} \langle\eta\rangle_k\, p_k\, \hat{h} \qquad \Delta I_{2k} = f_2 \frac{\lambda}{2} (p_k\, \hat{h})^2$$

$$\Delta I_{3k} = f_3 \frac{\lambda}{2} |p_k\, \hat{h}|^3 \qquad \Delta I_{4k} = f_3 \frac{\lambda}{2} \langle\eta\rangle_k\, (p_k\, \hat{h})^3 \qquad \Delta I_{5k} = f_3 \frac{\lambda}{2} \langle\mathcal{H}\rangle_k\, |p_k\, \hat{h}|^3$$

4.4 Approximative calculation of dipole down-feeds

For an off-momentum orbit following a dispersive periodic solution, all multipoles act like small dipoles (dipole down-feed) and thus contribute to the radiation integrals affecting emittance, damping partitions etc.

Since OPA knows only flat lattices, the mosts simple model treats each multipole as a small bending magnet of angle $\Delta\phi = x'_{\rm out} - x'_{\rm in}$, curvature $h = \Delta\phi/L$, with L the length of the element, and gradient $k = (n-1)\, b_n\, \bar{x}^{n-2}$ for $n \geq 2$, where the average horizontal offset is taken as the mean between offset before and after the multipole: $\bar{x} = (x_{\rm out} + x'_{\rm in})/2$. Same simple averaging is used for optical parameters like dispersion η and lattice invariant $\mathcal{H}$ to calculate the contribution to radiation integrals. Multipoles of zero length are ignored, of course.

This approximation is very rough and will not exactly reproduce results obtained by codes using more complete models based on calculation of the field perpendicular to the beam and its contribution to radiation integrals in both transverse dimensions. But it is helpful, for example, to detect possible loss of damping beyond some momentum deviation – which has to be checked then using a code like MAD or TRACY.

4.5 Calculation of geometric acceptance

A particle touching an elliptical beampipe of half axis a_x and a_y fulfills the condition

$$\left(\frac{|x_o| + \sqrt{A_x\beta_x}}{a_x}\right)^2 + \left(\frac{\sqrt{A_y\beta_y}}{a_y}\right)^2 = 1$$

x_o is the horizontal closed orbit, in the error free, perfectly linear lattice given by dispersion. $y_o = 0$ due to OPA's restriction on flat lattices, but generalization is straightforward. Betatron amplitudes are given by

$$A_x = \gamma_x(x - x_o)^2 + 2\alpha_x(x - x_o)(x' - x'_o) + \beta_x(x' - x'_o)^2$$

with the corresponding formula for y. Betatron amplitudes thus are given by the starting conditions of the tracked particle. For coupled motion at a given and constant ratio of betatron amplitudes κ, with

$$A = A_x + A_y \qquad A_y = \kappa A \qquad A_x = (1 - \kappa)A,$$

i.e. $\kappa = 0$ pure horizontal, $\kappa = 1$ pure vertical oscillation, the maximum total amplitude $\tilde{A}(\kappa)$ accepted by the lattice is the minimum of the limitations from all the elliptical apertures of the machine:

$$\sqrt{\tilde{A}(\kappa)} = \min_k \left(-p_k + \sqrt{p_k^2 - q_k}\right) \qquad \text{with}$$

$$p_k = \frac{a_{yk}^2|x_{ok}|\sqrt{1-\kappa}\sqrt{\beta_{xk}}}{n_k} \quad q_k = \frac{a_{yk}^2(x_{ok}^2 - a_{xk}^2)}{n_k} \qquad n_k = a_{yk}^2\beta_x(1-\kappa) + a_{xk}^2\beta_{yk}\kappa$$

The contour of the geometric acceptance in the (x, y) plane at the location of the track point (t) is thus given by

$$\tilde{x}(\kappa) = x_{ot} \pm \sqrt{(1-\kappa)\tilde{A}(\kappa)\beta_{xt}} \qquad \tilde{y}(\kappa) = \sqrt{\kappa\tilde{A}(\kappa)\beta_{yt}} \qquad \kappa = 0 \ldots 1$$

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